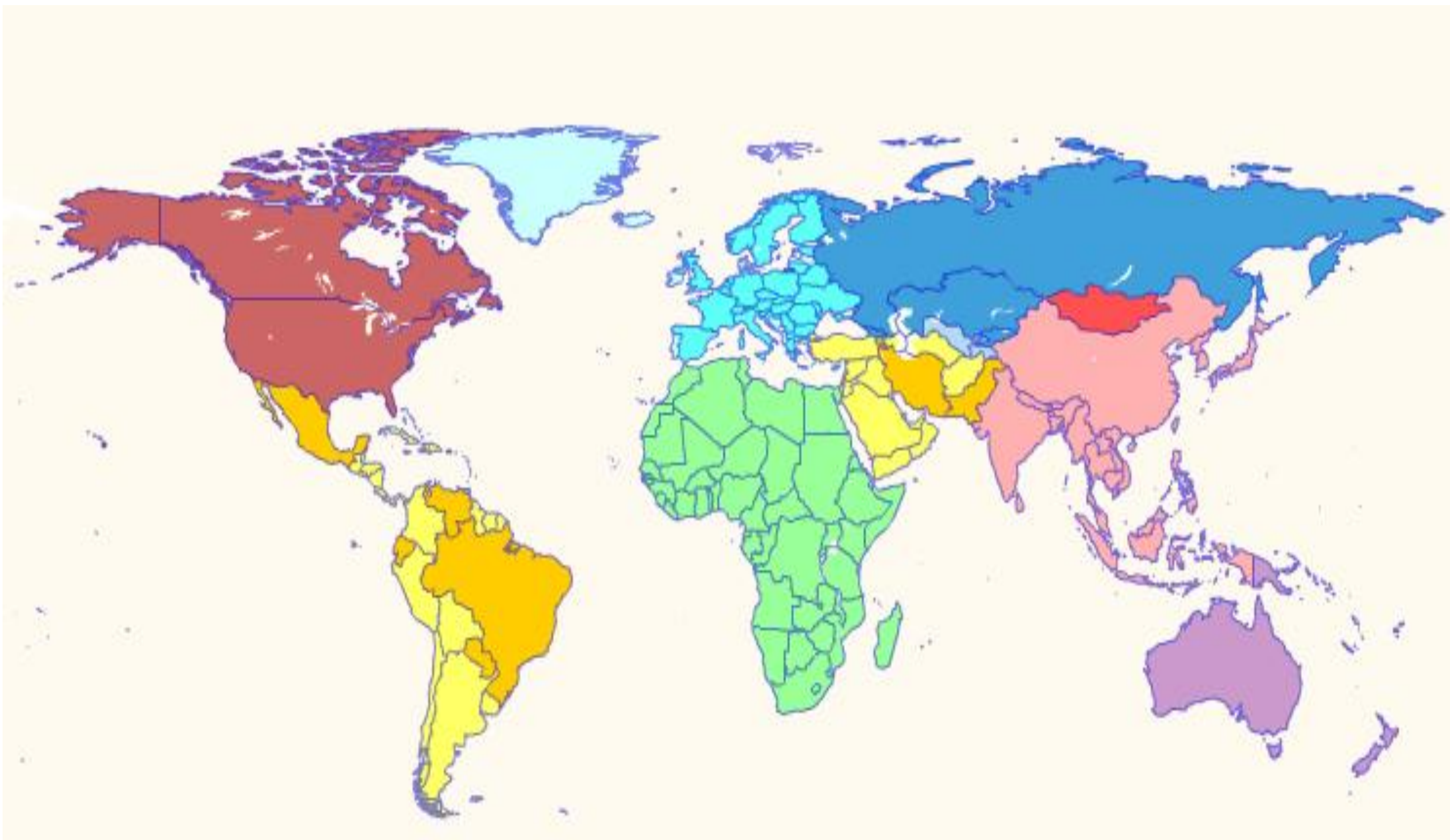
International Math Kangaroo Contest

Online Training
March 9, 2014
Instructor: Velian Pandeliev

Grade 7 - 8

International Math Kangaroo Contest (51 participating countries)



International Facts

- The contest began in 1991 in France and it runs every year.
- Open for students aged 6-18.
- Currently, there are 51 countries in the international association "Kangaroo Without Borders".
- Over 6,355,000 students participated worldwide in 2013.
- The first Canadian edition of the Math Kangaroo was in 2001 in Ottawa.

23 Locations Across Canada

Contest Information

Date: March 23, 2014 (Sunday)

Who can write: Students in grades 1-12

The Kangaroo math contest has **30** multiple-choice questions.

You will have **75** minutes to answer them all.

They are divided into three parts of **10** questions each:

Part A (easy) - correct answer is worth **3 points**

Part B (medium) - correct answer is worth **4 points**

Part C (hard) - correct answer is worth **5 points**

Questions left blank are worth 0 points. Wrong answers carry a penalty of -1 point. The maximum score is **150 points**. To avoid negative scores, everyone start with 30 points.

Calculators are not permitted.

The Response Form

International Contest - Game
“Math Kangaroo” Canada, 2013

SAMPLE Response form
Grade 3-4

Student’s Name: YOUR NAME   Grade: YOUR GRADE

E-mail: EMAIL ADDRESS   Phone: PHONE

Please, circle the correct answer:

1	A B C D E	**9**	A B C D E	**17**	A B C D E

Strategies

The Kangaroo math contest consists of 30 multiple-choice questions to be answered in 75 minutes.

That means you only have two and a half minutes for every question!

If you get stuck on a question, skip it, do the other ones and come back to it when you're sure you have time to try again.

Very few students finish the entire contest in the time allotted and answer every question correctly.

Do not be discouraged if you find you can't do some questions.

Remember, if you don't know the answer, don't guess! It's better to leave the answer blank than to risk losing 1 point if you guessed wrong.

This Session

In this session I will talk a bit about the contest and what you should expect.

Then I will give you 12 questions typical of the Grade 7-8 contest.

You will be presented with each question and you'll have about a minute to work on it independently and give me an answer in the poll on the right.

Then I will talk you through one possible solution.

Don't worry about copying down everything on the slides as they will be posted to the Math Kangaroo site after the session.

Please have pen and paper handy, and put your thinking caps on!

Question 1 (3 points)

A pedestrian crossing is marked using alternating 50 cm white and black stripes. If this crossing starts and ends with a white stripe and has a total of 8 white stripes, how wide is the crossing?

(A) 7 m	(B) 7.5 m	(C) 8 m	(D) 8.5 m	(E) 9 m

There are eight white stripes, and the first and last ones are both white.

That means that there are 7 black stripes between the 8 white stripes.

A total of 15 stripes, each 50 cm (0.5 m) wide

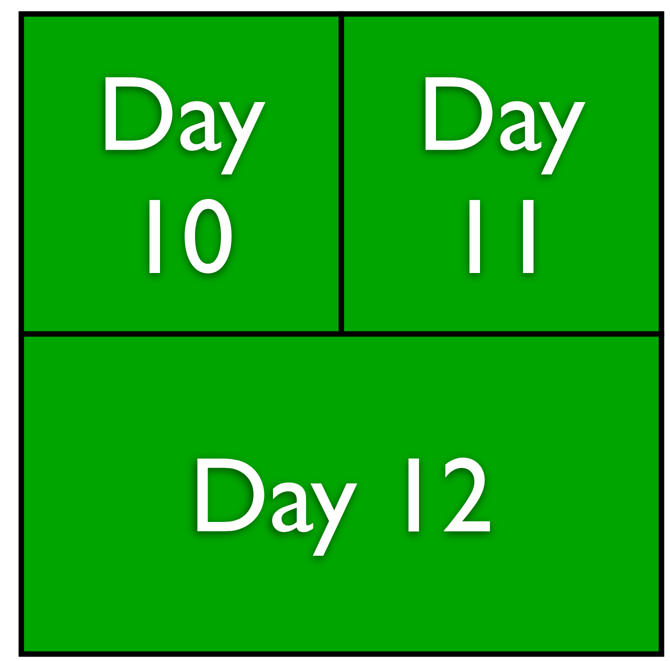
gives us a total of 15 x 0.5 = $\boxed{7.5 \text{ m}}$

Question 2 (3 points)

Lillies grow on the surface of a pond. Every day they cover twice the surface of the lake than they occupied the previous day. How long will it take the lillies to cover the entire surface of the lake if it takes them 10 days to cover a quarter of it?

(A) 12	(B) 15	(C) 20	(D) 25	(E) 30

How fast do the lillies grow? We're told that they double the surface area they cover every day. If it took ten days to cover a quarter of the lake, will it take ten more days to cover another quarter?



Nope, just one more day.

1/2 covered on day 11, the whole lake will be covered by Day 12

Question 3 (5 points)

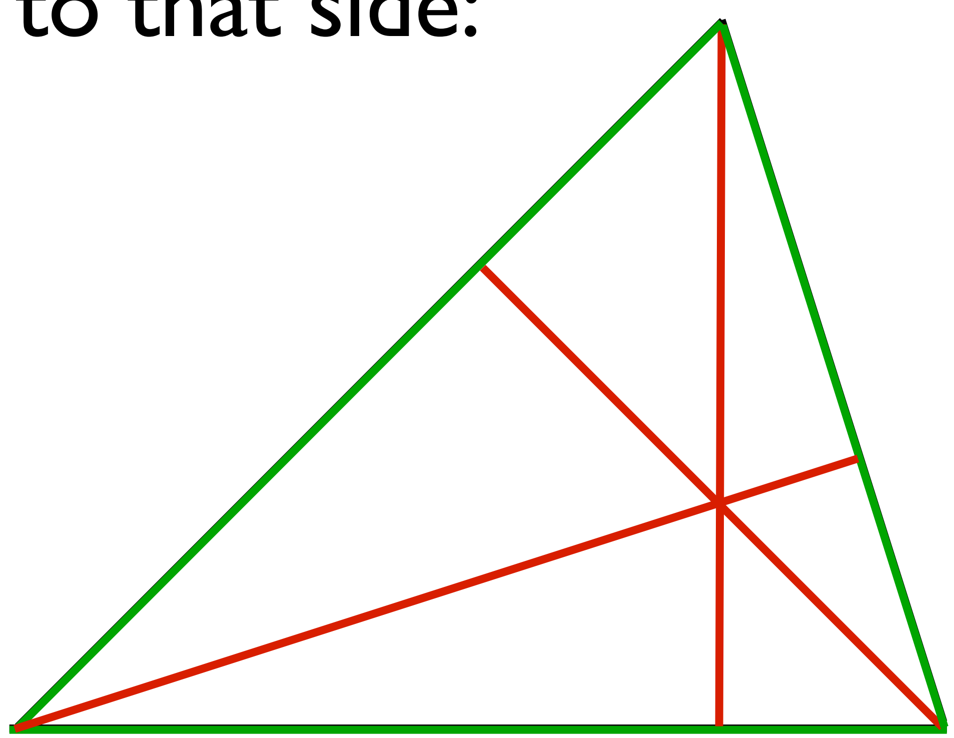
A triangle has a side of length 6 and a side of length 8. Which of the following cannot be its area?

(A) 20	(B) 24	(C) 19.1	(D) 25	(E) 5

What is the formula for the area of a triangle?

(base x height) / 2

Each of a triangle's three sides can serve as a base, and the corresponding height is the perpendicular line from the third vertex to that side:

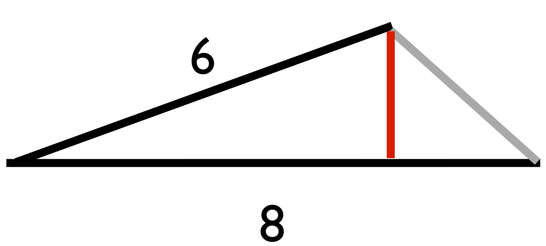


Question 3 (5 points)

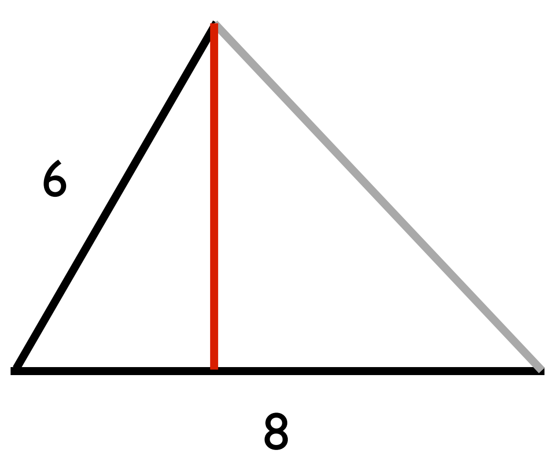
A triangle has a side of length 6 and a side of length 8. Which of the following cannot be its area?

(A) 20	(B) 24	(C) 19.1	(D) 25	(E) 5

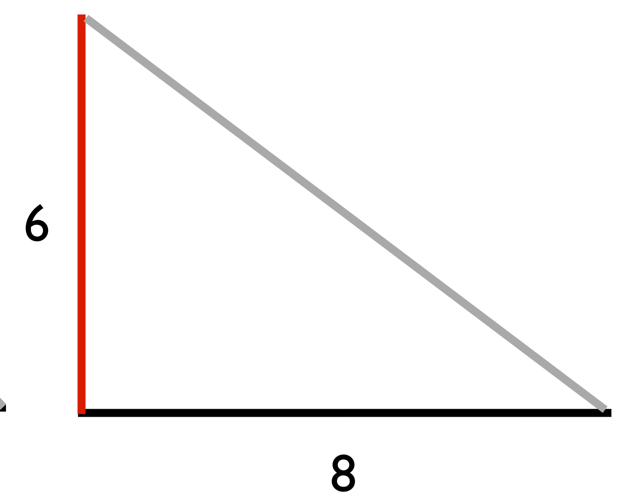
We have a triangle with sides 8 and 6.
Let's pick the 8 side as the base and see what our heights look like depending on the angle between the base and the 6 side.



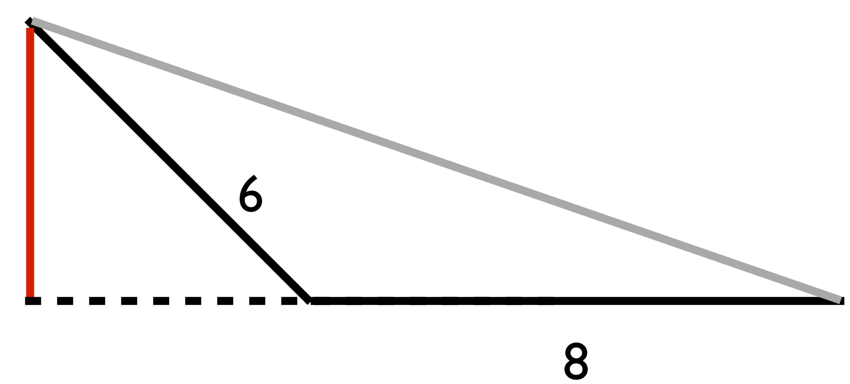
Small angle, small height, small area



As angle grows, height grows too but it's still < 6



When the angle is 90°, the height is exactly 6



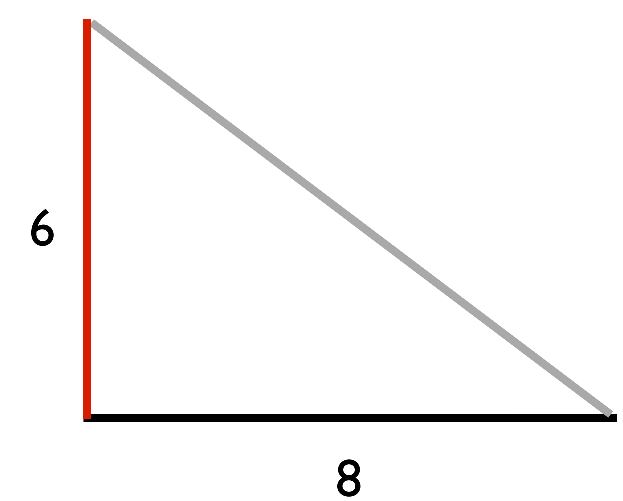
When the angle is obtuse, the height is < 6 again

Question 3 (5 points)

A triangle has a side of length 6 and a side of length 8. Which of the following <u>cannot</u> be its area?

(A) 20	(B) 24	(C) 19.1	(D) 25	(E) 5

We can have really small areas and really large areas depending on the angle between the sides.
Which triangle has the largest area?
The one with the largest height,
the right-angle triangle.



The largest possible area for a triangle with sides 6 and 8 is

6 x 8 / 2 = 24.

The only area listed in the answers that is definitely not possible is $\boxed{25}$

Question 4 (4 points)

Alice lies on Mondays, Wednesdays and Thursdays and tells the truth the rest of the week. Bob lies on Thursdays, Fridays and Sundays and tells the truth the rest of the week.
One day Alice said: "Today is Monday" and Bob confirmed "It's true, it is Monday." What day of the week is it?

(A) Sunday	(B) Monday	(C) Wednesday	(D) Thursday	(E) other

Alice said "It is Monday".
However, she's supposed to be lying on Mondays.
If it was really Monday, she would not have said so, therefore it is not Monday.
Then Bob confirms that it is Monday, meaning that Bob lied too.
What day of the week is it if Alice and Bob are both lying?
The only day when they are both lying is Thursday

Question 5 (5 points)

Basil and Peter were given math problems to solve during the break. Basil received 4 times as many problems as Peter. At the end of the break, it turned out that Basil and Peter solved the same number of problems. However, the percentage of Peter's problems that he solved was equal to the percentage of Basil's problems he didn't solve. What percentage of his problems did Peter solve?

(A) 40%	(B) 50%	(C) 60%	(D) 80%	(E) 90%

This problem has a relatively simple solution, which one of your peers discovered yesterday.

I have included my old solution at the end of this presentation.

It uses **variables** and equations in order to express relationships between unknown quantities. It's good to read if you're interested in algebra.

Question 5 (5 points)

Basil and Peter were given math problems to solve during the break. Basil received 4 times as many problems as Peter. At the end of the break, it turned out that Basil and Peter solved the same number of problems. However, the percentage of Peter's problems that he solved was equal to the percentage of Basil's problems he didn't solve. What percentage of his problems did Peter solve?

(A) 40%	(B) 50%	(C) 60%	(D) 80%	(E) 90%

Let's call the number of problems Peter solved **P**.

Since Basil solved the same number, he also solved **P**.

Now let's call the total number of problems Peter was supposed to solve **T**.

Basil was given four times as many, so 4 x **T** problems.

Question 5 (5 points)

What else do we know?

We know that the percentage (so the fraction) of problems solved by Peter is equal to the percentage (so the fraction) of problems not solved by Basil.

fraction of problems Peter solved $\dfrac{P}{T}$

fraction of problems Basil solved $\dfrac{P}{4 \times T}$ or $\dfrac{1}{4}\,\dfrac{P}{T}$

Look closely at the two fractions.

The fraction for Basil is actually 1/4 of the fraction for Peter.

This means that Peter's fraction is 4 times greater than Basil's!

We also know something else:

$$\underbrace{\frac{P}{T}}_{\text{fraction of problems Peter solved}} = \underbrace{\boxed{100\% - \frac{P}{4 \times T}}}_{\text{fraction of problems Basil did not solve}}$$

Question 5 (5 points)

$$\text{fraction of problems Peter solved} \quad \frac{P}{T} = \boxed{100\% - \frac{P}{4 \times T}} \quad \text{fraction of problems Basil did not solve}$$

But Peter's fraction is 4 times greater than Basil's fraction:

$$4 \times \boxed{\text{\% Basil solved}} = 100\% - \boxed{\text{\% Basil solved}}$$

$$5 \times \boxed{\text{\% Basil solved}} = 100\%$$

Basil solved 1/5 of 100%, which is 20% of his problems,

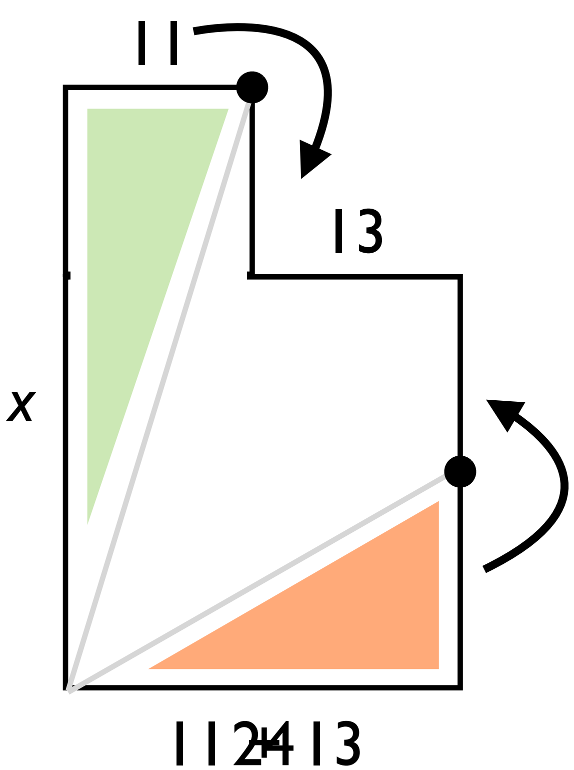
and Peter solved the same percentage that Basil didn't solve

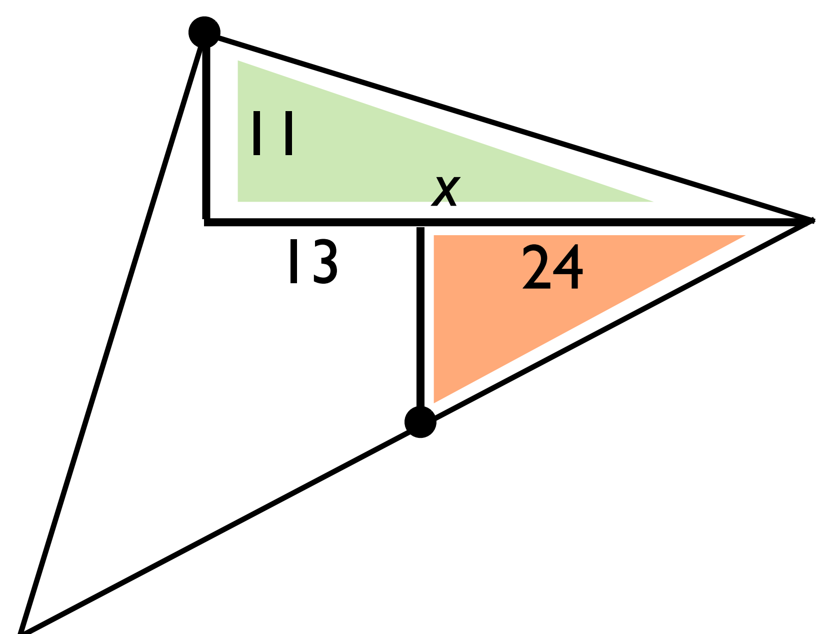
(4/5 of 100%) which is $\boxed{80\%}$

Question 6 (4 points)

The figure on the left shows a shape consisting of two rectangles. The lengths of two segments are marked 11 and 13. The shape is then cut into three parts and they are rearranged into a triangle, as shown in the right-hand figure. What is the length of the segment marked with *x*?

(A) 24	(B) 26	(C) 36	(D) 37	(E) 39





$$x = 13 + 24 = \boxed{37}$$

Question 7 (4 points)

Yahtzee is a popular game in which players take turns rolling five standard dice (1 to 6, opposite sides add up to 7). On her turn, Toni rolled at least three 6's. What is the probability that her total was greater than 25?

(A) 5/12	(B) 1/3	(C) 1/4	(D) 1/6	(E) 7/18

What are probabilities and why are they expressed as fractions? The probability of rolling a 3 on a six-sided die is 1 in 6, or 1/6: in 1/6 of all possible rolls, 3 will be the result.
In this problem, we are trying to get more than 25 points from five dice, three of which we already know have shown 6's.
That means that the remaining two dice have to add up to more than 25 - (3 x 6) = 25 - 18 = 7.
If we roll 2 dice, what is the probability of getting a total of **8 or more**?

Question 7 (4 points)

Yahtzee is a popular game in which players take turns rolling five standard dice (1 to 6, opposite sides add up to 7). On her turn, Toni rolled at least three 6's. What is the probability that her total was greater than 25?

(A) 5/12	(B) 1/3	(C) 1/4	(D) 1/6	(E) 7/18

Each die has 6 possibilities, so two dice have 6 x 6 = 36 possibilities.
How many of those score 8?
2 and 6, 3 and 5, 4 and 4, 5 and 3, 6 and 2 = 5 ways
How many score 9?
3 and 6, 4 and 5, 5 and 4, 6 and 3 = 4 ways
For 10, it's 4 and 6, 5 and 5, 6 and 4 = 3 ways
For 11, it's 5 and 6, 6 and 5 = 2 ways
For 12, it's only 6 and 6 = 1 way
There are a total of 5 + 4 + 3 + 2 + 1 = 15 ways to score 8 or more using 2 dice.

Question 7 (4 points)

Yahtzee is a popular game in which players take turns rolling five standard dice (1 to 6, opposite sides add up to 7). On her turn, Toni rolled at least three 6's. What is the probability that her total was greater than 25?

(A) 5/12	(B) 1/3	(C) 1/4	(D) 1/6	(E) 7/18

So, out of 36 possibilities, 15 yield 8 or more,

meaning that in 15 out of every 36 cases, Toni rolled more than 25.

What is that as a probability?

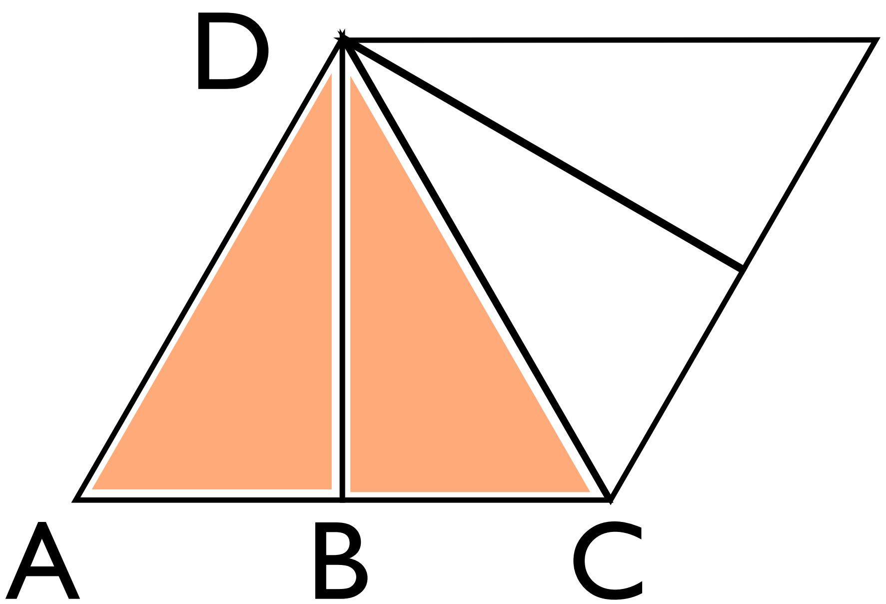
15 out of 36 expressed as a fraction is 15/36,

which reduces to $\boxed{5/12}$

Question 8 (4 points)

The rhombus in the figure is made up of four congruent right-angled triangles. How many degrees is the acute angle of the rhombus?

(A) 30°	(B) 45°	(C) 60°	(D) 75°	(E) ???



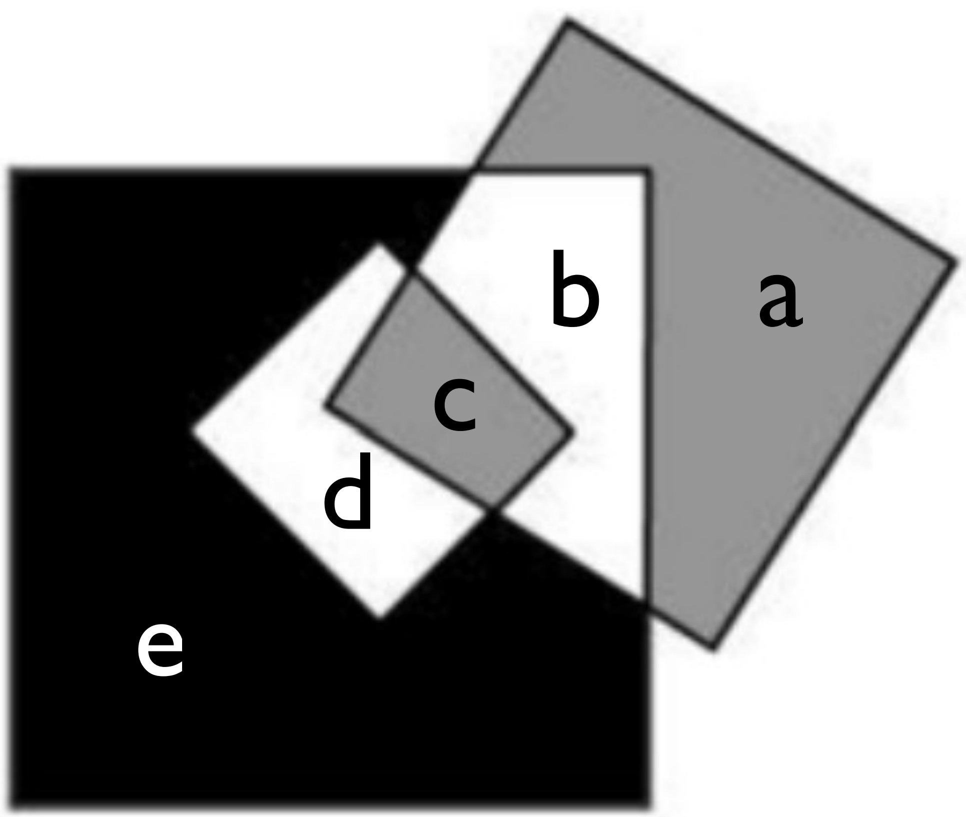
We need to know two things:
- the definition of a rhombus
(a parallelogram with four equal sides)
- the definition of "congruent triangles"
(triangles whose sides and angles are equal)

Since triangle ABD is congruent with triangle CBD, AD = DC.
From the figure being a rhombus, we know that AC = AD.
What that means is that in triangle ACD, AC = AD = DC.
That means that triangle ACD is equilateral.
An equilateral triangle has equal sides and angles, and one of them is the acute angle of the rhombus, which is 180° / 3 = $\boxed{60°}$

Question 9 (5 points)

The figure shows a square with side 3 cm inside a square with side 7 cm and another square with side 5 cm which intersects the first two squares. What is the difference between the black region and the total area of the grey regions?

(A) 10 cm^2	(B) 12 cm^2	(C) 15 cm^2	(D) 0 cm^2	(E) ???



black: $e = 7 \times 7 - d - c - b$

$= 49 - d - c - b$

grey: a + c is the sum of the areas of the two small squares, minus the white areas (b and d) and minus c, which we have counted twice, once as part of each square.

$a + c = 5x5 + 3x3 - d - c - b$

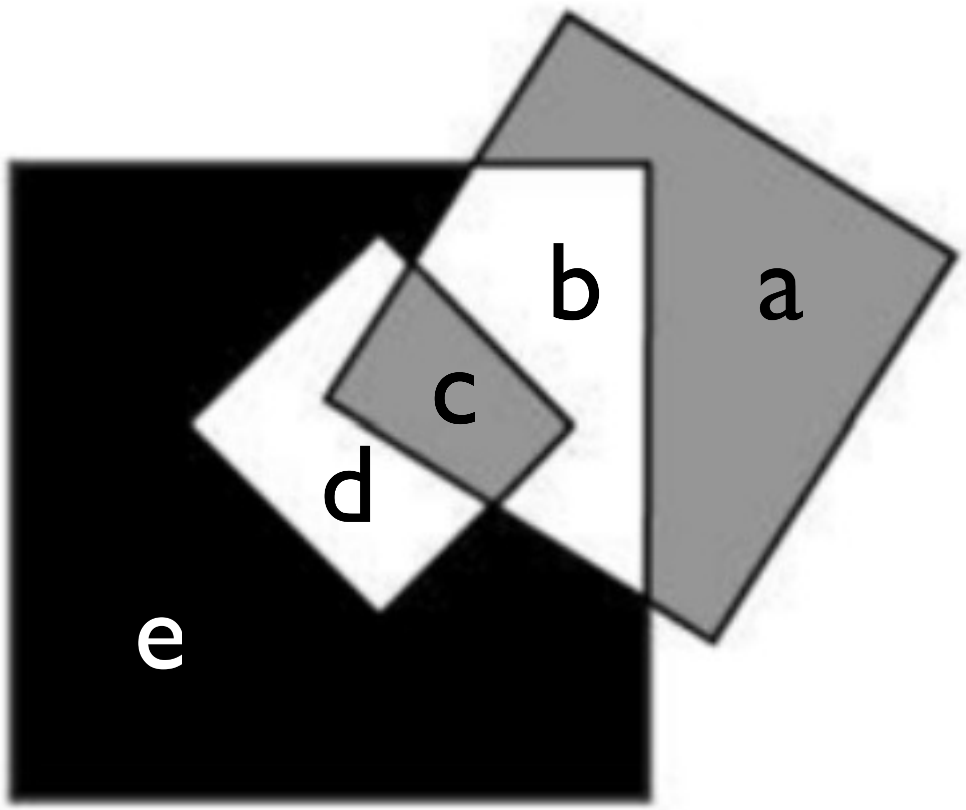
$= 25 + 9 - d - c - b$

$= 34 - d - c - b$

Question 9 (5 points)

The figure shows a square with side 3 cm inside a square with side 7 cm and another square with side 5 cm which intersects the first two squares. What is the difference between the black region and the total area of the grey regions?

(A) 10 cm^2	(B) 12 cm^2	(C) 15 cm^2	(D) 0 cm^2	(E) ???



black: e = 49 - d - c - b

grey: a + c = 34 - d - c - b

We are looking for the difference between the two, and both have the same terms (-d -b -c).

e - (a + c) = 49 - 34 (-d-c-b) - (-d-b-c)

Those subtract to 0, giving us:

e - (a + c) = 49 - 34

= $\boxed{15\ cm^2}$

Question 10 (4 points)

What is the last digit of the sum $1^2 + 2^2 + 3^2 + ... + 2014^2$?

(A) 3	(B) 4	(C) 5	(D) 6	(E) 9

Since we are only looking at final digits, we should check what the last digit of the square of each number from 1 to 10 is.

number	squared	digit
1	1	1
2	4	4
3	9	9
4	16	6
5	25	5
6	36	6
7	49	9
8	64	4
9	81	1
10	100	0

The sum of the first ten squares' last digits is

$1+4+9+6+5+6+9+4+1+0 = 45$

The sum of the first ten will end in 5, and the sum of the first twenty will end in 0.

Since 2000 is divisible by 20, the sum of the first 2000 squares ends in 0.

Question 10 (4 points)

What is the last digit of the sum $1^2 + 2^2 + 3^2 + \ldots + 2014^2$?

(A) 3	(B) 4	(C) 5	(D) 6	(E) 9

number	**squared**	**digit**
1	1	1
2	4	4
3	9	9
4	16	6
5	25	5
6	36	6
7	49	9
8	64	4
9	81	1
10	100	0

SS up to	**ends in**
2000	0
2010	5
2011	5 + 1 = 6
2012	6 + 4 = _0
2013	0 + 9 = 9
2014	9 + 6 = _5

The last digit of the sum of squares from 1 to 2014 is

$\boxed{5}$

Question 11 (5 points)

7 years ago Evie's age was a multiple of 8 and in eight years, her age will be a multiple of 7. 8 years ago Ralph's age was a multiple of 7 and in 7 years his age will be a multiple of 8. Both Evie and Ralph are less than 100 years old.

Which of the following statements is true:

A) Ralph is 2 years older than Evie.

B) Ralph is 1 year older than Evie.

C) Ralph and Evie are the same age.

D) Ralph is 1 year younger than Evie.

E) Ralph is 2 years younger than Evie.

Question 11 (5 points)

7 years ago Evie's age was a multiple of 8 and in eight years, her age will be a multiple of 7. 8 years ago Ralph's age was a multiple of 7 and in 7 years his age will be a multiple of 8. Both Evie and Ralph are less than 100 years old.

Evie: 7 years ago, her age was a multiple of 8, say it was 8 x A.
Then, today Evie is 8 x A + 7 years old.
In 8 years, she will be 8 x A + 8 + 7 years old, which is
8 x (A + 1) + 7 and should be a multiple of 7.
Since 7 is a multiple of 7,
8 x (A + 1) also has to be a multiple of 7.
Since 8 is not a multiple of 7, A + 1 must be a multiple of 7.
A can be 6, 13, 20, etc. but 8 x A + 7 has to be less than 100,
so A has to be 6.
That means that today Evie is 8 x 6 + 7 = 55 years old.

Question 11 (5 points)

7 years ago Evie's age was a multiple of 8 and in eight years, her age will be a multiple of 7. 8 years ago Ralph's age was a multiple of 7 and in 7 years his age will be a multiple of 8. Both Evie and Ralph are less than 100 years old.

Ralph: 8 years ago, Ralph's age was a multiple of 7, say 7 x B.
Today, Ralph is 7 x B + 8 years old.
In 7 years, Ralph will be 7 x (B + 1) + 8 years old.
and his age will have to be a multiple of 8.
7 x (B + 1) is a multiple of 8,
so B + 1 must be a multiple of 8,
so B is 7, 15, etc., but 7 x B + 8 has to be less than 100,
so B = 7.
Ralph is 7 x 7 + 8 = 49 + 8 = 57 years old, meaning that

$\boxed{\text{Ralph is 2 years older than Evie}}$

Lesson: Algebra (reuniting broken parts)

When we perform simple operations like 2 x 3 or 5 + 9, we are not "doing math". We are performing **arithmetic**. Arithmetic is cool and useful, but it requires that we know every number in the equation.

Sometimes, as you will find in this session, we are trying to find quantities we don't know by performing arithmetic operations. The techniques to do that are from a different branch of math called **algebra**, from Arabic: "the reunion of broken parts".

Here's how it works:

Let's say we don't know how many eggs I ate this morning, but we do know that I could eat three times as many and still have 6 eggs left in my 12-egg carton.

What we have is an equality, with an unknown:

$$3 \times \boxed{???} + 6 = 12$$

number of eggs Velian ate this morning

Lesson: Algebra (reuniting broken parts)

The important thing here is that this equality must always be respected - we are not allowed to do anything that makes the equality untrue, otherwise we lose the only information we have. Also, so we don't have to draw a square every time, I'm going to use the letter E instead of "number of eggs Velian ate".

$$3 \times E + 6 = 12$$

Our goal when solving equations like this is always to get the unknown term (in our case E) on its own on one side of the scale. What are we allowed to do without breaking the equality? For one thing, we can add or subtract the same thing to both sides:

$$3 \times E + 6 \boxed{-6} = 12 \boxed{-6}$$

Lesson: Algebra (reuniting broken parts)

$$3 \times E \quad + 6 \quad \boxed{-\ 6} \quad = \quad 12 \quad \boxed{-\ 6}$$

This helps, because +6 - 6 is just 0, and we know how to do 12 - 6.

$$3 \times E \quad = \quad 6$$

Other things we can do is multiply both sides by the same number, or divide both sides by the same number **as long as it is not 0**. On the left, we have 3 x E, but we want just E. So, we divide by 3:

$$3 \times E \quad \boxed{\div\ 3} \quad = \quad 6 \quad \boxed{\div\ 3}$$

$$E \quad = \quad 2$$

That's it, we're done. We found our unknown by performing allowed arithmetic to both sides of an equation.
These techniques can be used on **any** equation!

Question 5 (5 points) (with variables)

Basil and Peter were given math problems to solve during the break. Basil received 4 times as many problems as Peter. At the end of the break, it turned out that Basil and Peter solved the same number of problems. However, the percentage of Peter's problems that he solved was equal to the percentage of Basil's problems he didn't solve. What percentage of his problems did Peter solve?

(A) 40%	(B) 50%	(C) 60%	(D) 80%	(E) 90%

Percentages are just parts of a whole expressed as a fraction over100. 80% is the same as 80/100 or 8/10 or 4/5.

If the percentage of Peter's problems that he solved is equal to the percentage of Basil's problems that he did not solve, then the fraction of solved over all for Peter will be equal to the fraction of unsolved over all for Basil.

Question 5 (5 points) (with variables)

Basil and Peter were given math problems to solve during the break. Basil received 4 times as many problems as Peter. At the end of the break, it turned out that Basil and Peter solved the same number of problems. However, the percentage of Peter's problems that he solved was equal to the percentage of Basil's problems he didn't solve. What percentage of his problems did Peter solve?

(A) 40%	(B) 50%	(C) 60%	(D) 80%	(E) 90%

My solution uses **variables** in order to express relationships between unknown quantities.

A more detailed explanation of the principles we can use and solve for variables is at the end of this presentation.

For now, follow along and trust me that I'm allowed to do the things I'm doing :)

Question 5 (5 points) (with variables)

Basil and Peter were given math problems to solve during the break. Basil received 4 times as many problems as Peter. At the end of the break, it turned out that Basil and Peter solved the same number of problems. However, the percentage of Peter's problems that he solved was equal to the percentage of Basil's problems he didn't solve. What percentage of his problems did Peter solve?

(A) 40%	(B) 50%	(C) 60%	(D) 80%	(E) 90%

Let's call the number of problems Peter solved **P**.

Since Basil solved the same number, he also solved **P**.

Now let's call the total number of problems Peter was supposed to solve **T**.

Since Basil solved four times as many, he solved 4 x **T** problems.

Question 5 (5 points) (with variables)

What else do we know?

We know that the percentage (so the fraction) of problems solved by Peter is equal to the percentage (so the fraction) of problems not solved by Basil.

What does this equality look like?

fraction of problems Peter solved $\dfrac{P}{T}$

fraction of problems Basil solved $\dfrac{P}{4 \times T}$

$$\frac{P}{T} = \boxed{\frac{4 \times T}{4 \times T} - \frac{P}{4 \times T}}$$

fraction of problems Peter solved

all problems assigned to Basil

fraction of problems Basil solved

fraction of problems Basil did not solve

Question 5 (5 points) (with variables)

$$\frac{P}{T} = \frac{4 \times T}{4 \times T} - \frac{P}{4 \times T}$$

My goal is to work out the relationship between **P** and **T**.

We can multiply by 4 x **T** on both sides to eliminate the fractions:

$$\frac{P \boxed{\times 4 \times \cancel{T}}}{\cancel{T}} = \frac{4 \times T \cancel{\boxed{\times 4 \times T}}}{\cancel{4 \times T}} - \frac{P \cancel{\boxed{\times 4 \times T}}}{\cancel{4 \times T}}$$

$$4 \times P = 4 \times T - P$$

Then, to get just **P** on one side and just **T** on the other, we add **P** to both sides:

$$4 \times P + P = 4 \times T - P + P$$

$$5 \times P = 4 \times T$$

Question 5 (5 points) (with variables)

$$5 \times \mathbf{P} \quad = \quad 4 \times \mathbf{T}$$

We still don't know what **P** or **T** are, but we know how they relate to each other.
For Basil, **P** represents the number of problems he solved and 4 x **T** represents the number of problems he was assigned.
This means that Basil solved 1/5 of his assigned problems, which as a percentage is 20%.
If Basil solved 20% of his problems, that means he did not solve 80% of his problems,
and we know from the original question that Peter solved the same percentage that Basil did not solve,
so the percentage of problems solved by Peter was $\boxed{80\%}$

International Math Kangaroo Contest

Thank you!
See you on March 23!