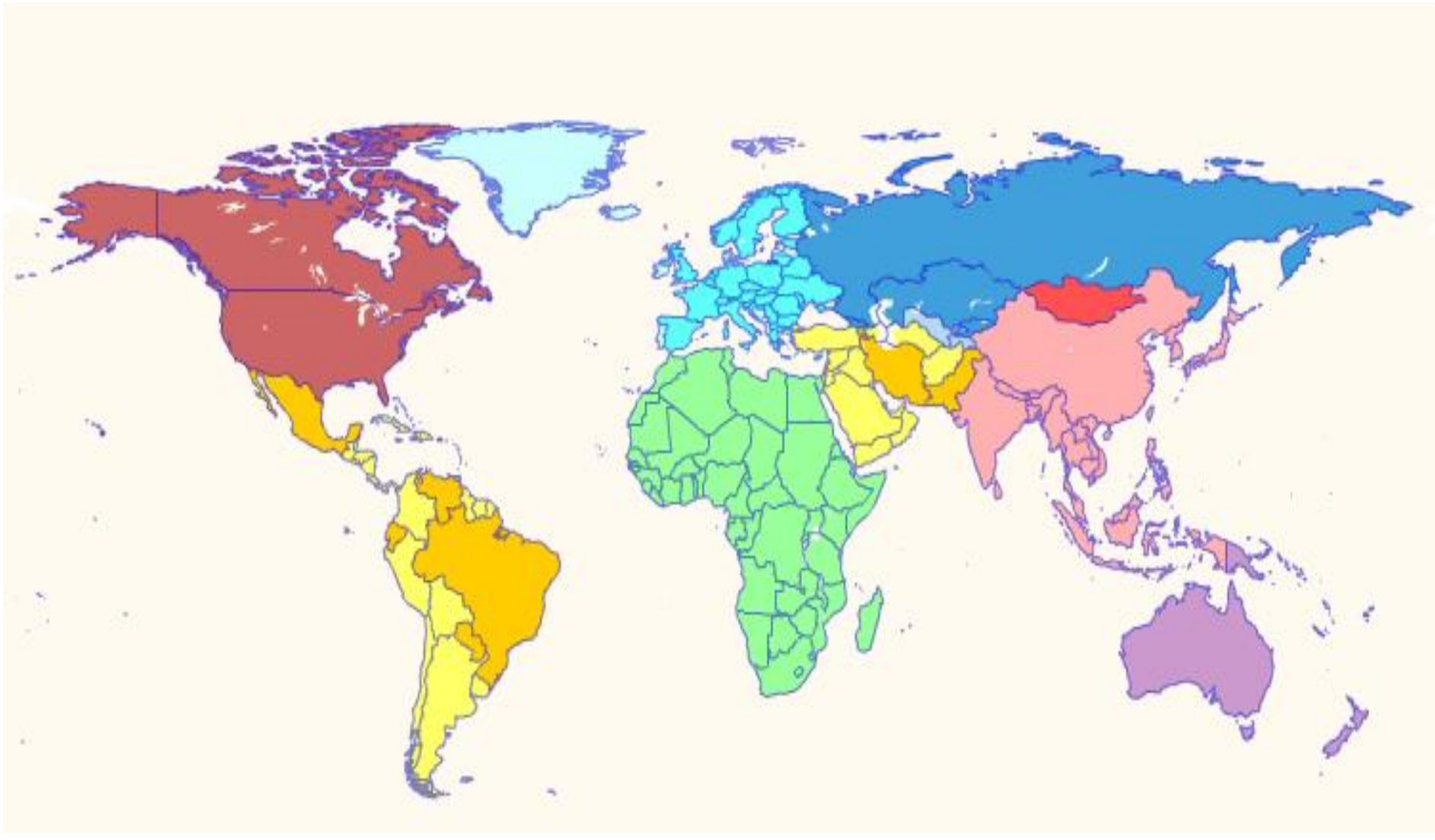
International Math Kangaroo Contest

Online Training
March 8/9, 2014
Instructor: Velian Pandeliev

Grade 5 - 6

International Math Kangaroo Contest (51 participating countries)



International Facts

- The contest began in 1991 in France and it runs every year.
- Open for students aged 6-18.
- Currently, there are 51 countries in the international association "Kangaroo Without Borders".
- Over 6,355,000 students participated worldwide in 2013.
- The first Canadian edition of the Math Kangaroo was in 2001 in Ottawa.

23 Locations Across Canada

Contest Information

Date: March 23, 2014 (Sunday)

Who can write: Students in grades 1-12

The Kangaroo math contest has **30** multiple-choice questions.

You will have **75** minutes to answer them all.

They are divided into three parts of **10** questions each:

Part A (easy) - correct answer is worth **3 points**

Part B (medium) - correct answer is worth **4 points**

Part C (hard) - correct answer is worth **5 points**

Questions left blank are worth 0 points. Wrong answers carry a penalty of -1 point. The maximum score is **150 points**. To avoid negative scores, everyone start with 30 points.

Calculators are not permitted.

The Response Form

International Contest - Game
"Math Kangaroo" Canada, 2013

SAMPLE Response form
Grade 3-4

Student's Name: YOUR NAME Grade: YOUR GRADE

E-mail: EMAIL ADDRESS Phone: PHONE

Please, circle the correct answer:

1 A B C D E	**9** A B C D E	**17** A B C D E

Strategies

The Kangaroo math contest consists of 30 multiple-choice questions to be answered in 75 minutes.

That means you only have two and a half minutes for every question!

If you get stuck on a question, skip it, do the other ones and come back to it when you're sure you have time to try again.

Very few students finish the entire contest in the time allotted and answer every question correctly.

Do not be discouraged if you find you can't do some questions.

Remember, if you don't know the answer, don't guess! It's better to leave the answer blank than to risk losing 1 point if you guessed wrong.

This Session

In this session I will talk a bit about the contest and what you should expect.

Then I will give you 11 questions typical of the Grade 5-6 contest.

You will be presented with each question and you'll have about a minute to work on it independently and give me an answer in the poll on the right.

Then I will talk you through one possible solution.

Don't worry about copying down everything on the slides as they will be posted to the Math Kangaroo site after the session.

Please have pen and paper handy, and put your thinking caps on!

Question 1 (3 points)

The oldest elf in the Elven Kingdom, Thranduil, is 2626 years old. The youngest, Elrohir, is only 26. How many times is Thranduil older than Elrohir?

(A) 11	(B) 100	(C) 101	(D) 1000	(E) 1001

There are two ways to do this.
One is using plain old long division.
It is not always taught in schools, but it is very important,
for instance when you're writing a math contest in which calculators are not allowed.
Let's divide 2626 by 26.

$$
\begin{array}{r}
101 \\
26\overline{)2626} \\
-26 \\
\hline
026 \\
-26 \\
\hline
0
\end{array}
$$

101

Question 1 (3 points)

The oldest elf in the Elven Kingdom, Thranduil, is 2626 years old. The youngest, Elrohir, is only 26. How many times is Thranduil older than Elrohir?

(A) 11	(B) 100	(C) 101	(D) 1000	(E) 1001

Before we move on, there is actually a simpler way to figure this out.

Let's look at the number 2626. It looks like it should be easy for us to see how it relates to 26.

One way to find out is to represent 2626 as a sum of two or more numbers that are easily divided by 26.

For example:

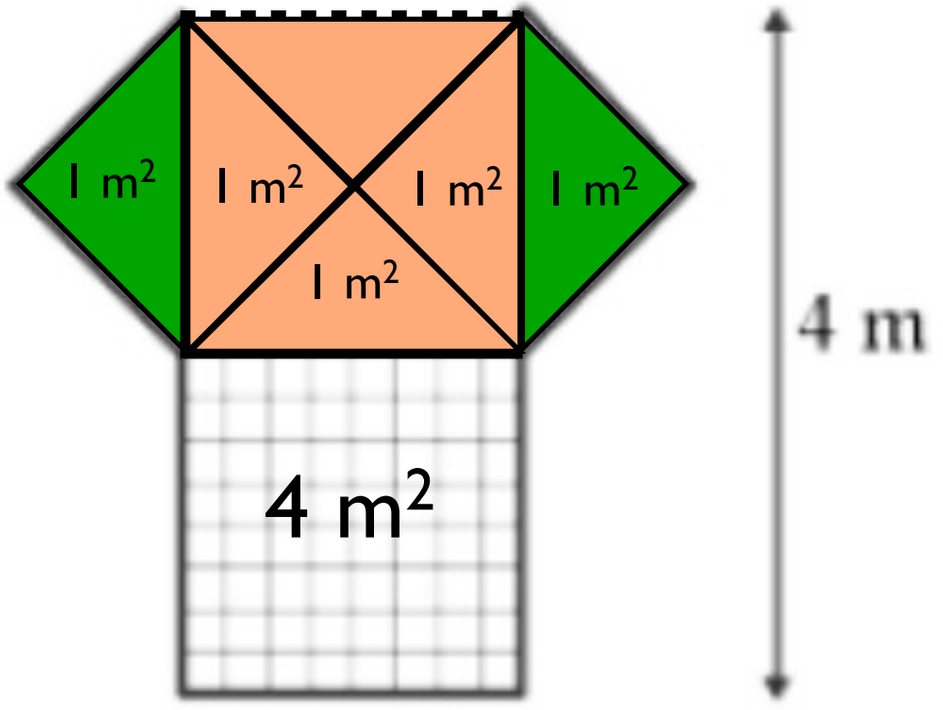
2626 = 2600 + 26

100 times 26, plus 1 times 26, also gives us **101**

Question 2 (4 points)

The figure shows a polygon, drawn to scale, such that the distance from the highest point to the base is 4 m. What is the area of the polygon?

(A) 9 m^2	(B) 8.25 m^2	(C) 8.5 m^2	(D) 9.5 m^2	(E) 10 m^2



We need to split the figure up into shapes whose areas we can find.

The bottom half of the shape is a 2 m x 2 m square, so its area is 4 m^2.

On the top we have five triangles that we can see are identical.

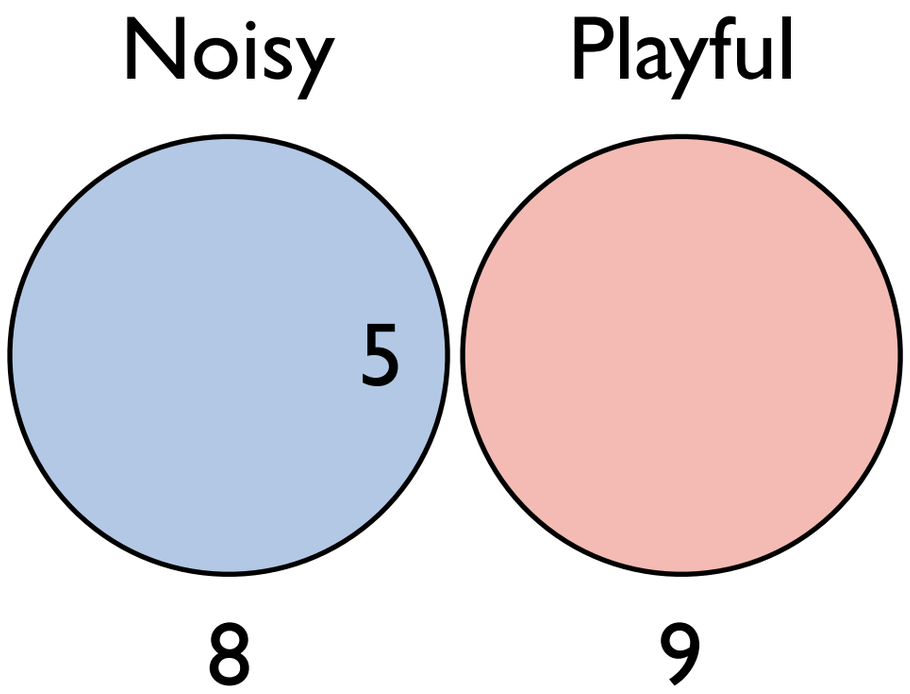
From the top square, each triangle looks like it's 1/4 of the area of the square. So each triangle has an area of 1 m^2.

The total area of the shape is $4 + 5 \times 1 = \boxed{9\ m^2}$

Question 3 (3 points)

Twelve puppies are playing in the meadow. Exactly eight of them are noisy, and nine are playful. How many puppies are both noisy and playful?

(A) None	(B) 3	(C) 4	(D) 5	(E) 8



If each puppy is only noisy or only playful, that makes 8 + 9 = 17

But we only have 12, meaning we have counted some puppies twice - once as playful and once as noisy.

How many? Well, as many as we are over: 17 - 12.

So five puppies are both noisy and playful.

That figure is called a Venn diagram, by the way, and it's very useful when dealing with problems like that.

5 puppies

Question 4 (4 points)

Alan, the youngest member, left the basketball team. How did the average age of the players change?

A) It increases.
B) It stays the same.
C) It decreases.
D) It may increase or decrease depending on Alan's age
E) It may increase or decrease depending on the age of the other players.

To solve this problem, we need to understand what the average of several numbers is. That's hard to do when we don't have any numbers to work with.

The average is the sum of all numbers, divided by the number of numbers we have.

For example, the average of the numbers 3, 4, 7 and 10 is $(3 + 4 + 7 + 10) \div 4 = 24 \div 4 = 6$.

Question 4 (4 points)

Alan, the youngest member, left the basketball team. How did the average age of the players change?

The average of 3, 4, 7 and 10 is 6.

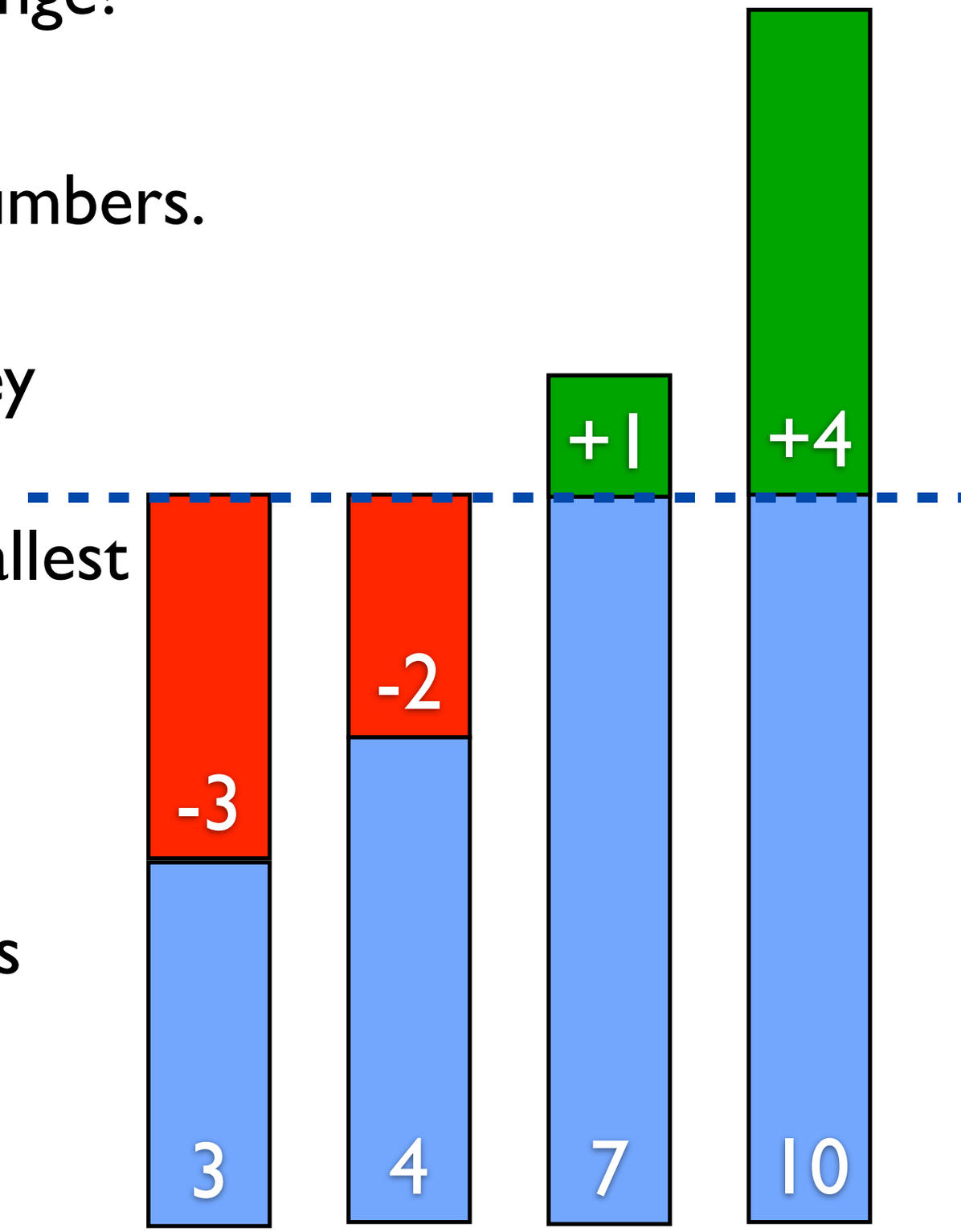
Notice that 6 is not one of the numbers.

Rather, it is a representation of what the numbers would be if they were all the same.

It's definitely greater than the smallest number and it's smaller than the largest number.

Notice something?

3 and 5 together are less than 6 as much as 7 and 10 together are greater than 6.



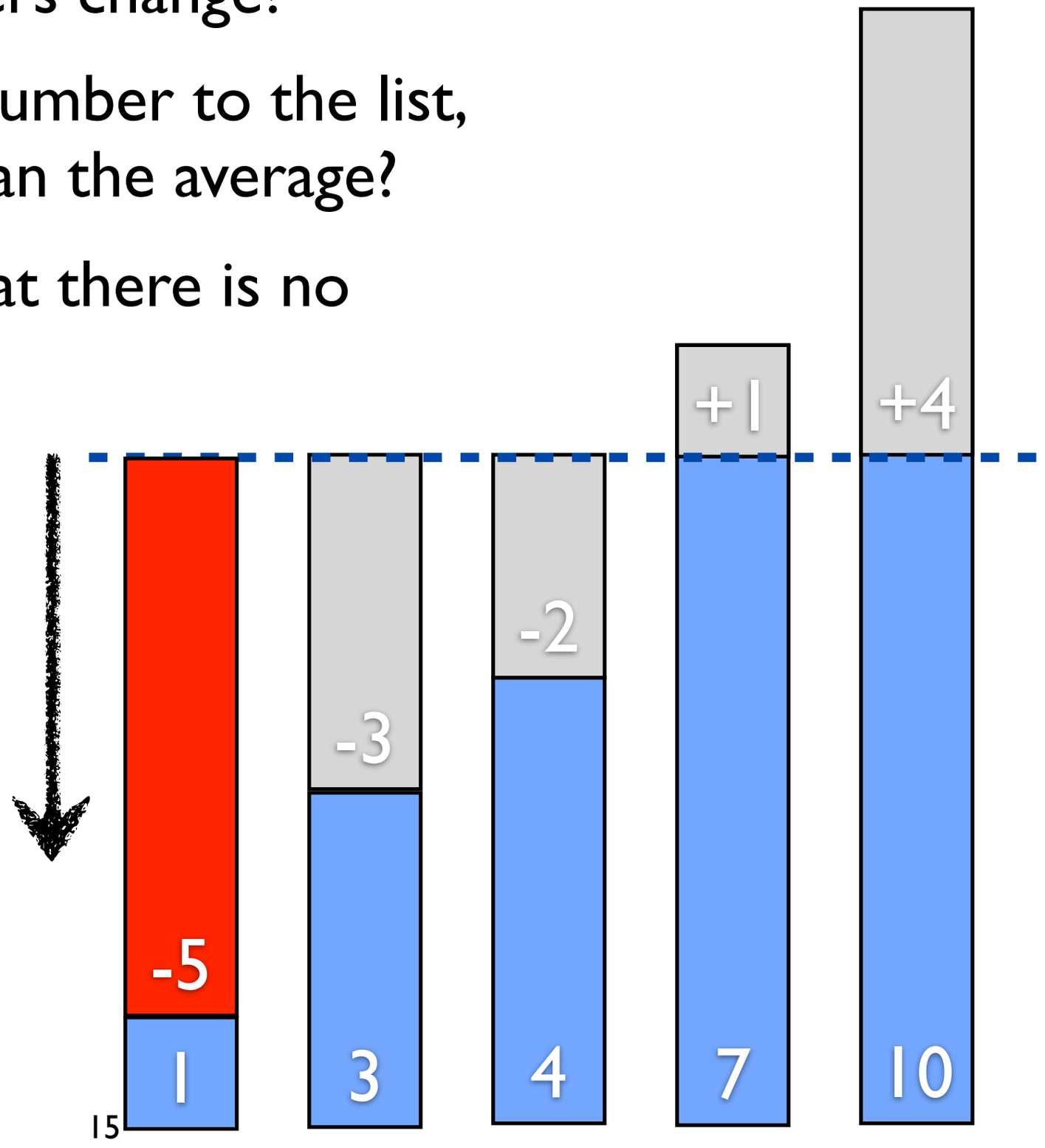
Question 4 (4 points)

Alan, the youngest member, left the basketball team. How did the average age of the players change?

What happens if we add a number to the list, and we know it's smaller than the average?

Well, it will have a deficit that there is no corresponding surplus for.

This is going to bring the whole average down until the bars above and below have equalized again.



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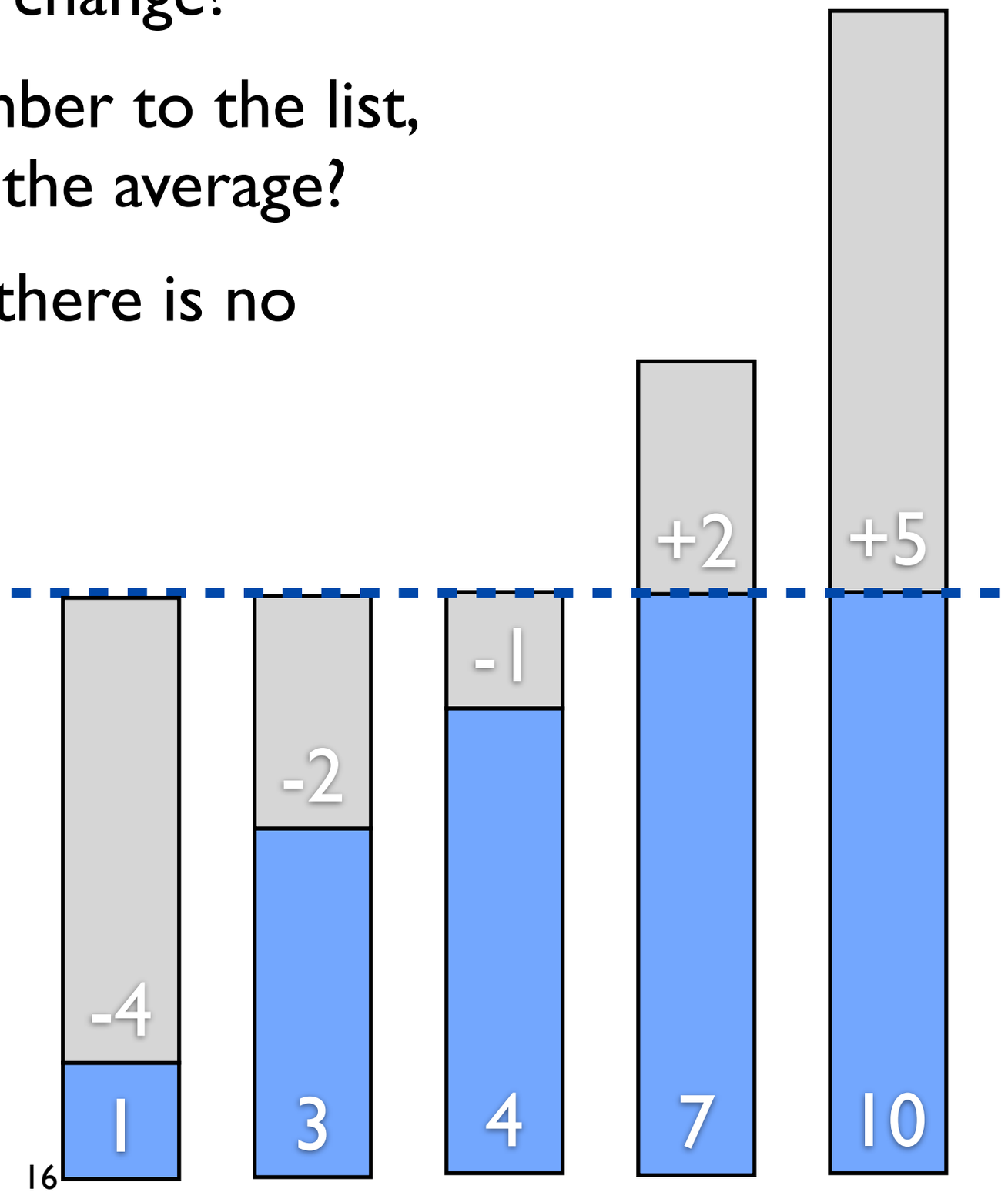
Alan, the youngest member, left the basketball team. How did the average age of the players change?

What happens if we add a number to the list, and we know it's smaller than the average?

Well, it will have a deficit that there is no corresponding surplus for.

This is going to bring the whole average down until the bars above and below have equalized.

The new average is
(1 + 3 + 4 + 7 + 10) ÷ 5
= 5, lower than before, because we added a term that was less than the average.



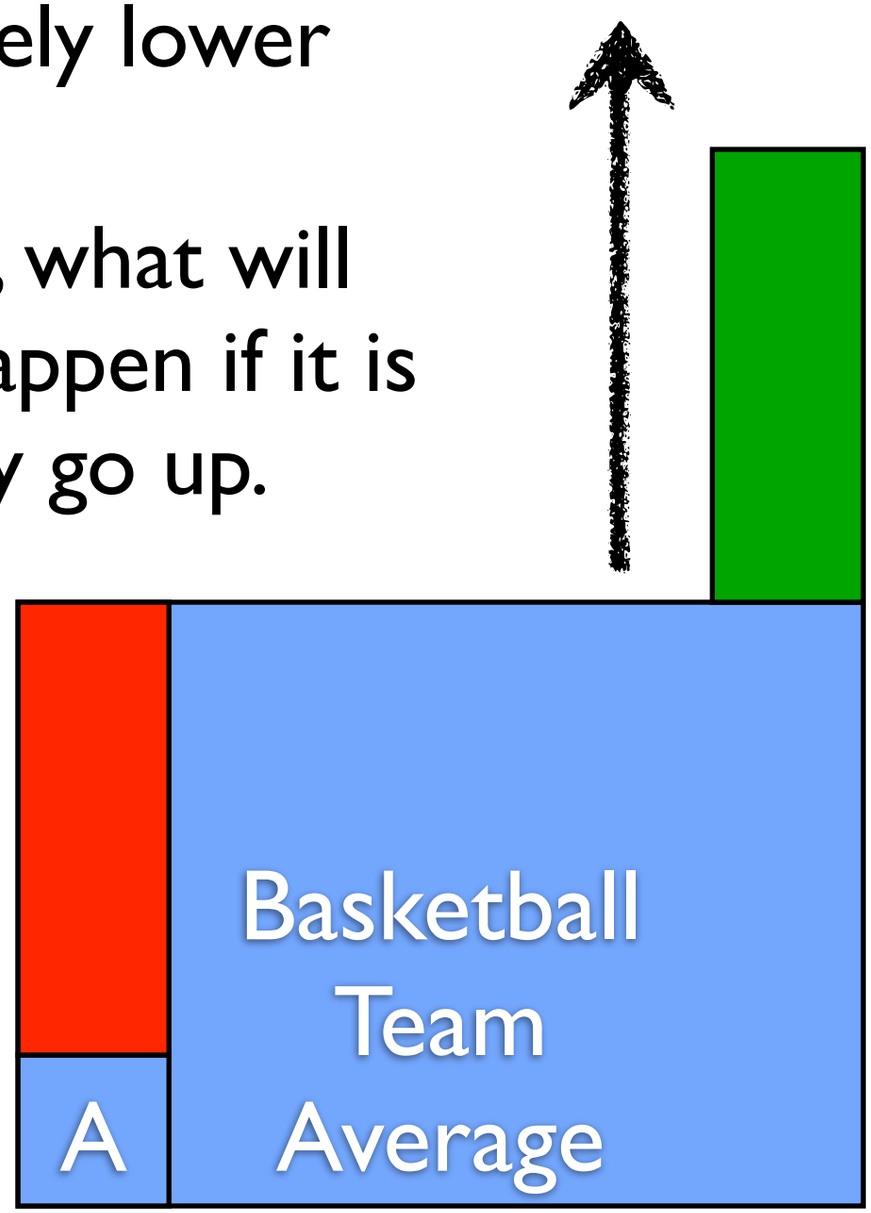
Question 4 (4 points)

Alan, the youngest member, left the basketball team. How did the average age of the players change?

We are finally able to answer our original question.

Alan is the youngest, so his age is definitely lower than the average.

If a lower than average term is removed, what will happen is the opposite of what would happen if it is being added: the average would definitely go up.



The average age increases.

Question 5 (4 points)

What is the last digit of the following product:

1 x 3 x 5 x 7 x 9 x 11 ... x 2007 x 2009 x 2011 x 2013

(A) 1	(B) 3	(C) 5	(D) 7	(E) 9

Not every big long piece of arithmetic can or should be calculated in full to get the answer.

Obviously, we cannot multiply all these numbers together in 2 minutes. We also don't need to.

Notice that one of the numbers is 5. 5 is a very special number when it comes to final digits, because no matter what you multiply it by, its last digit can only be one of two things:

5 x (any **odd** number)	ends in 5
5 x (any **even** number)	ends in 0

Question 5 (4 points)

What is the last digit of the following product:

1 x 3 x 5 x 7 x 9 x 11 ... x 2007 x 2009 x 2011 x 2013

(A) 1	(B) 3	(C) 5	(D) 7	(E) 9

Because the order of terms in multiplication doesn't matter (commutative property), the expression above is really:

5 x (1 x 3 x 7 x ... x 2011 x 2013)

Is the second term even or odd?

Well, for it to be even, at least one of the terms should be even or divisible by two.

However, we can see that all those terms are odd.

Since 5 times any odd number ends in 5, the final digit of the product will be 5

Question 6 (4 points)

July 13, 2010 was a Tuesday. What is the next year in which July 13 will also be a Tuesday?

(A) 2011	(B) 2016	(C) 2017	(D) 2018	(E) 2021

To solve this problem we need to know a few things about the calendar:

In a regular year, there are 365 days.

That's 365 ÷ 7 = 52 weeks, with a remainder of 1.

What does that mean?

It means that if a particular date is a Tuesday one year, it will be Wednesday the following year.

Then we have leap years. A leap year happens when the year is divisible by 4 (or by 400 if it ends in 00).

Leap years have 366 days, meaning they advance the day of the week not by 1, but by 2 days for all dates after Feb. 29.

Question 6 (4 points)

July 13, 2010 was a Tuesday. What is the next year in which July 13 will also be a Tuesday?

(A) 2011	(B) 2016	(C) 2017	(D) 2018	(E) 2021

2010	regular (+1)	Tuesday
2011	regular (+1)	Wednesday
2012	**leap (+2)**	Friday
2013	regular (+1)	Saturday
2014	regular (+1)	Sunday
2015	regular (+1)	Monday
2016	**leap (+2)**	Wednesday
2017	regular (+1)	Thursday
2018	regular (+1)	Friday
2019	regular (+1)	Saturday
2020	**leap (+2)**	Monday
2021	regular (+1)	Tuesday

2021

Question 7 (3 points)

Six of King Arthur's knights are sitting around a round table. Knights who are sitting next to each other are enemies and knights who are not are friends. We want to choose two knights who are friends for a dangerous quest.

How many such pairs are there?

(A) 3	(B) 6	(C) 9	(D) 12	(E) 18

Each knight has two neighbours, meaning he has two enemies.
He can't go with himself, and he can't go with an enemy.
That leaves each knight with three possible friends to accompany him on the quest.
The number of possible pairs is 6 x 3 = 18.

Question 7 (3 points)

Six of King Arthur's knights are sitting around a round table. Knights who are sitting next to each other are enemies and knights who are not are friends. We want to choose two knights who are friends for a dangerous quest.

How many such pairs are there?

(A) 3	(B) 6	(C) 9	(D) 12	(E) 18

However! Let's pretend that Sir Lancelot is sitting at the table, looking across at his friend Sir Gawain.

We have counted Lancelot + Gawain as a possible pair.

But Sir Gawain is also friends with Sir Lancelot, meaning we have also counted Gawain + Lancelot as part of the 18.

In fact, each pair has been counted twice, so we need to divide the total number by 2.

9 pairs

Question 8 (5 points)

In the following cryptoarithmetic puzzle, each letter represents a digit (different letters represent different digits and the same letters represent the same digit).
What is the value of the sum A + B + C + D + E?

(A) 9	(B) 10	(C) 11	(D) 12	(E) 20

```
  BDCE
+BDAE
-----
 AECBE
```

These puzzles are hard, but once you get one of the letters, the rest will unravel quickly.

Usually we start with the units column, because there we don't have to worry about a carry.

We see that `E + E = _E` (something ending in `E`).

The only digit for which that is possible is 0, so `E = 0`

Question 8 (5 points)

In the following cryptoarithmetic puzzle, each letter represents a digit (different letters represent different digits and the same letters represent the same digit).
What is the value of the sum A + B + C + D + E?

(A) 9	(B) 10	(C) 11	(D) 12	(E) 20

```
  BDC0
+BDA0
-----
A0CB0
```

Now, lets look at the first two columns.

One thing we should know that when adding two four-digit numbers, even the largest five-digit result starts with 1.

Example: 9999 + 9999 = 19998.

That means that no matter what B is, A = 1.

In fact, any carry value from the sum of two digits is going to be either 0 or 1.

Question 8 (5 points)

In the following cryptoarithmetic puzzle, each letter represents a digit (different letters represent different digits and the same letters represent the same digit).
What is the value of the sum A + B + C + D + E?

(A) 9	(B) 10	(C) 11	(D) 12	(E) 20

```
  BDC0
+BD10
------
10CB0
```

So, `B + B + (0 or 1) = 10`.

Since 10 is an even result, and `B + B` is even, that means that we don't have a carry from the hundreds column.

`B + B = 10`

So `B = 5`.

Question 8 (5 points)

In the following cryptoarithmetic puzzle, each letter represents a digit (different letters represent different digits and the same letters represent the same digit).
What is the value of the sum A + B + C + D + E?

(A) 9	(B) 10	(C) 11	(D) 12	(E) 20

```
  5DC0
+5D10
------
10C50
```

Now then in the tens column, `C + 1 = _5`.
This result could be 5 or 15, but then `C` would have to be greater than 9, and that's not possible.
So, `C + 1 = 5`,
Meaning `C = 4`.

Question 8 (5 points)

In the following cryptoarithmetic puzzle, each letter represents a digit (different letters represent different digits and the same letters represent the same digit).
What is the value of the sum A + B + C + D + E?

(A) 9	(B) 10	(C) 11	(D) 12	(E) 20

```
  5D40
+5D10
-----
 10450
```

Finally, in the hundreds column:

`D + D + (carry) = _4`

There is no carry from the tens column, so:

`D + D = _4`

The result could be 4 or 14, but the hundreds column does not give a carry to the thousands column, so `D + D = 4`

That is only possible if `D = 2`.

Question 8 (5 points)

In the following cryptoarithmetic puzzle, each letter represents a digit (different letters represent different digits and the same letters represent the same digit).
What is the value of the sum A + B + C + D + E?

(A) 9	(B) 10	(C) 11	(D) 12	(E) 20

```
  5240
+5210
-----
 10450
```

We have solved the puzzle.
The digits were:

```
A = 1
B = 5
C = 4
D = 2
E = 0
```

```
 BDCE
+BDAE
-----
AECBE
```

So their sum is 1 + 5 + 4 + 2 + 0 = 12

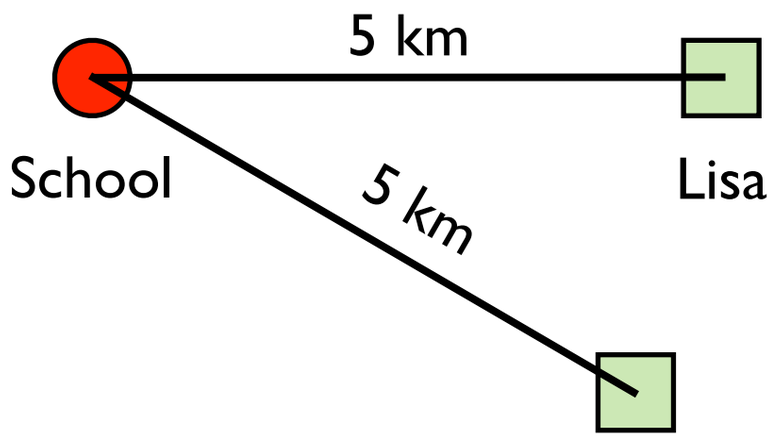
Question 9 (5 points)

Lisa and Nils go to the same school. Lisa lives 5 km away from the school and Nils lives 3 km away. How far away do Lisa and Nils live from each other?

(A) 2 km	(B) 2 km or 8 km	(C) 8 km	(D) Between 2 km and 8 km.	(E) Can't determine.

Let's draw the school and Lisa's house since we know how far apart they are.

Are we sure that's where it is? It could be here:



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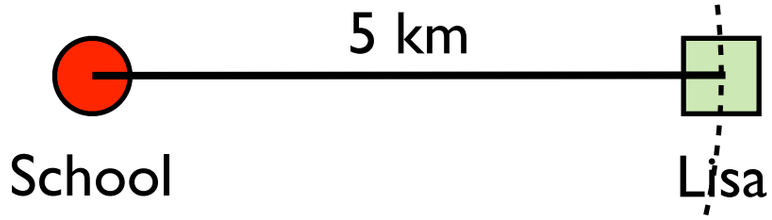
(A) 2 km	(B) 2 km or 8 km	(C) 8 km	(D) Between 2 km and 8 km.	(E) Can't determine.

All the points that are 5 km away from the school actually form a circle with radius 5 km.

Lisa's house can be anywhere on that circle.

Let's say it's where we drew it originally, but remember the circle.

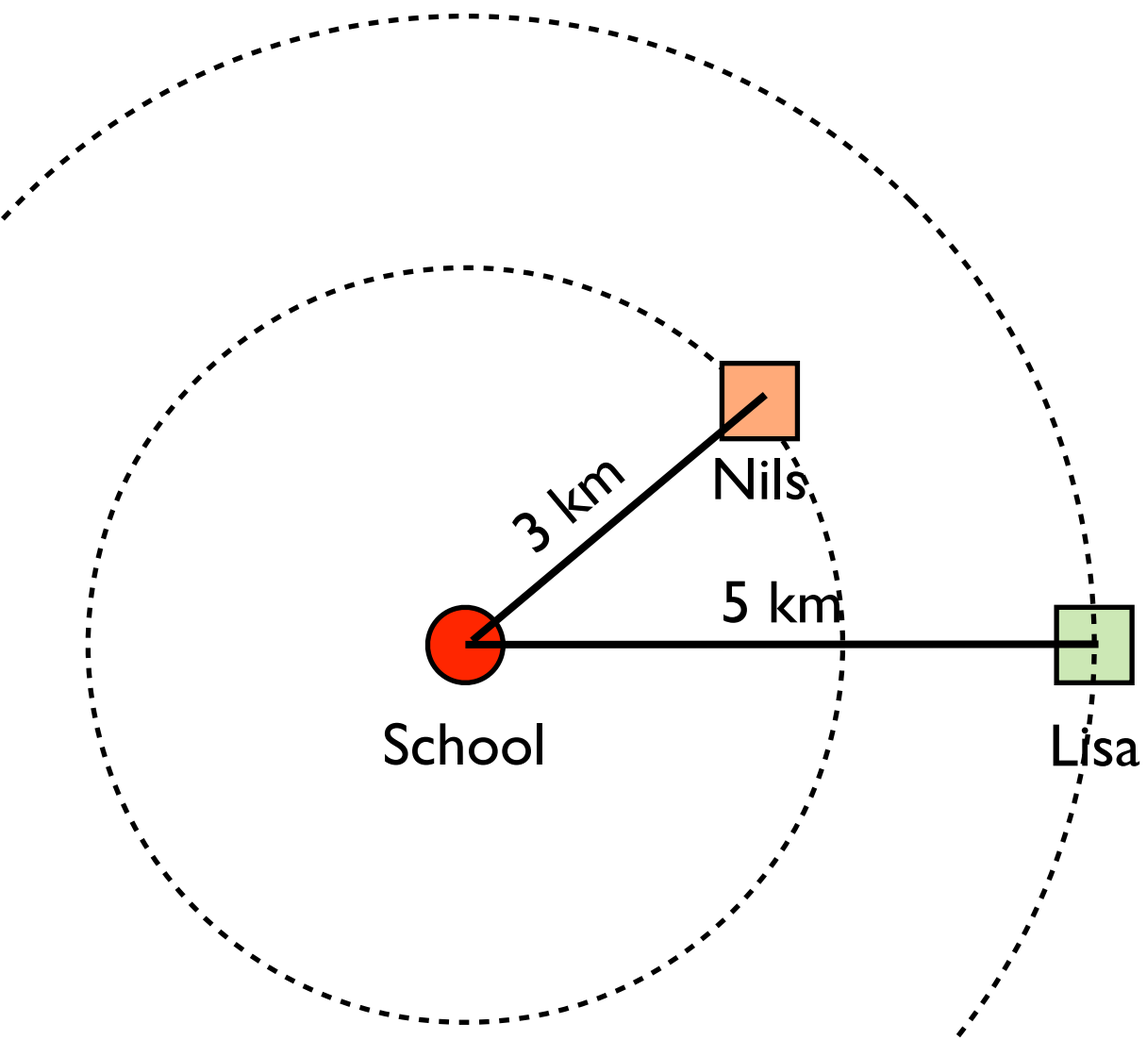


Question 9 (5 points)

Lisa and Nils go to the same school. Lisa lives 5 km away from the school and Nils lives 3 km away. How far away do Lisa and Nils live from each other?

Now, where does Nils live?

He lives somewhere on a different circle, with its centre at the school and a radius of 3 km.



Since both their houses can be anywhere, we cannot say for sure how far apart they live.

But can we say what is the closest they can live?

Question 9 (5 points)

Lisa and Nils go to the same school. Lisa lives 5 km away from the school and Nils lives 3 km away. How far away do Lisa and Nils live from each other?

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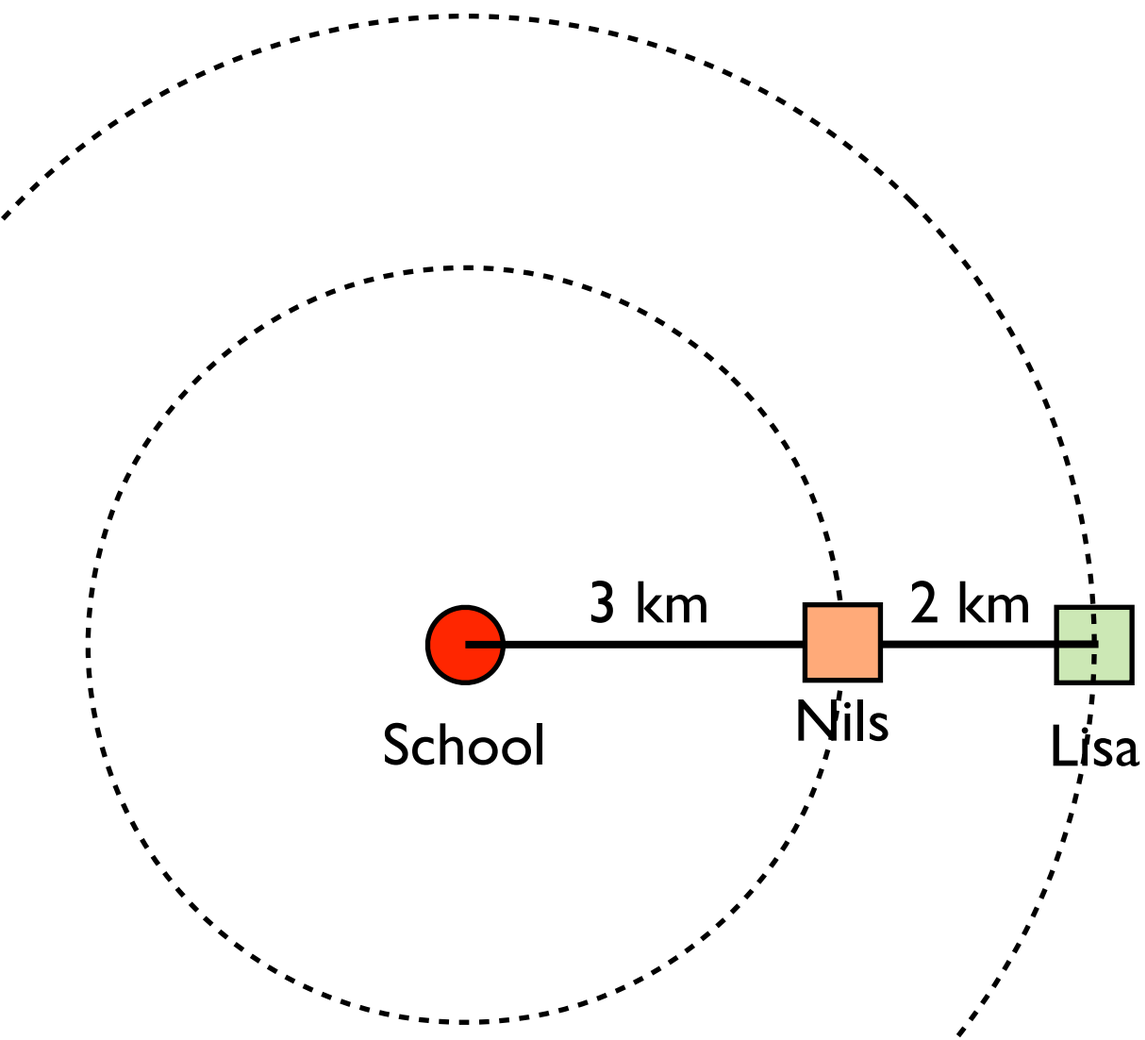

Since both their houses can be anywhere, we cannot say for sure how far apart they live.

But can we say what is the closest they can live?

It turns out that if Nils's house is on the same line as Lisa's, the distance between them is the smallest, only 2 km.

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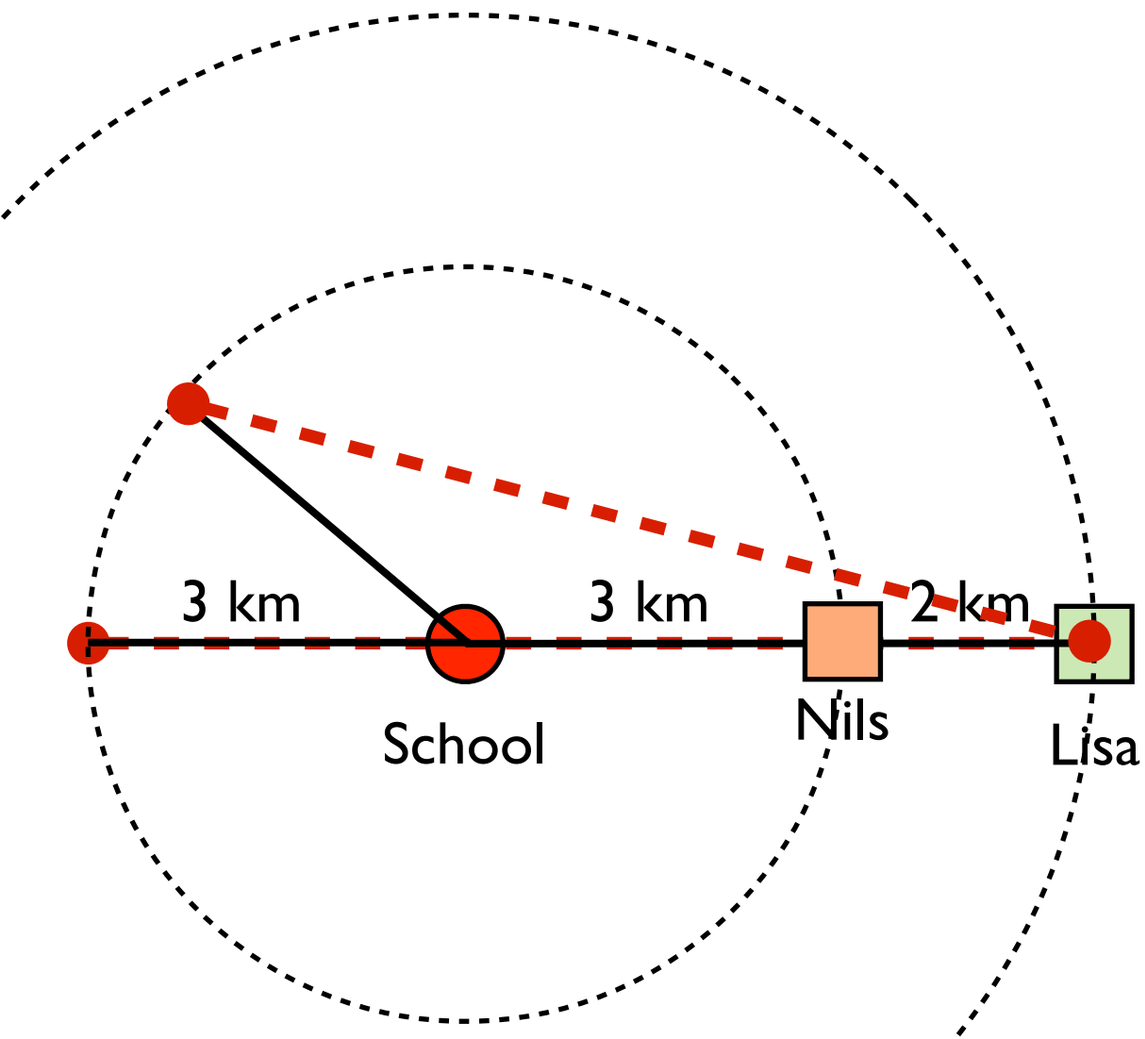

As Nils's house goes around the circle the distance will increase until he is at the opposite end of Lisa's house, then it will start to decrease again.

But he can't be any further than the furthest his circle will allow, which is 8 km away.

between 2 and 8 km

Question 10 (5 points)

How many six-digit numbers contain the group of digits "2014" when written down?

(A) 10	(B) 50	(C) 90	(D) 100	(E) 280

This is a counting question. The most important thing is to make sure we don't miss any cases.

We have a six-digit number, and we know that four of those digits are going to form the number 2014.

Given that, we only have three types of numbers:

2	**0**	**1**	**4**	?	?
?	**2**	**0**	**1**	**4**	?
?	?	**2**	**0**	**1**	**4**

Let's count the possibilities for each type of number.

Each spot can have the digits 0-9 unless it's at the beginning of the number, when it cannot be 0.

Question 10 (5 points)

How many six-digit numbers contain the group of digits "2014" when written down?

(A) 10	(B) 50	(C) 90	(D) 100	(E) 280

2	**0**	**1**	**4**	?	?

10 possibilities for the first digit and
10 possibilities for the second,
so a total of 10 x 10 = **100**

?	**2**	**0**	**1**	**4**	?

10 possibilities for the last digit but only 9 possibilities for the first,
so a total of 9 x 10 = **90**

?	?	**2**	**0**	**1**	**4**

10 possibilities for the second digit but only 9 possibilities for the first,
so a total of 9 x 10 = **90**

All the numbers, then, are 100 + 90 + 90 = 280

Question 11 (4 points)

How many zeroes are there at the end of the product
1 x 2 x 3 x 4 x 5 ... x 23 x 24 x 25?

(A) 6	(B) 5	(C) 4	(D) 3	(E) 2

Zeroes in this product can come from one of three places:

- multiplying by a number that ends in 0
- multiplying a number that ends in 5 by an even number
- multiplying a number that ends in 25 by a multiple of 4 (that gives us two zeroes!)

Case	**Numbers**	**Zeroes**
ends in 0	10(x8)x20(x12)	2
ends in 5	5(x2)x15(x6)	2
ends in 25	25 (x 4)	2

6

International Math Kangaroo Contest

Thank you!
See you on March 23!